Mathematical modeling of microalgae-mineralization-human structure within the environment regeneration system for the biosphere compatible city

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Abstract

Modern fast growing megacities and urban agglomerations exert extremely high environmental load. One of the biggest problems of megalopolises is the human organic waste disposal. Under current technology, waste utilization requires enormous extent of sewer networks and large areas of treatment plants, which deform the natural landscapes and expensive to operate. Organization of local recycling of organic waste is crucially important at creation of new urban objects satisfying the principle of biosphere compatibility. In addition, it should be noted, that with the rising of building tallness in modern cities the use of centralized water systems leads to excessive overruns of electricity by pumping equipment, therefore the local effluent regeneration systems is the optimal solution for skyscrapers. Making a start from successful design experience of closed ecological life support systems, which were initially created for long-duration space flights, our research implements the idea to realize closed circle for flows of substances and energy for utilization of human waste and improving air quality within residential building. We focus on small volume treatment facilities designed for 20-30 people. The main difference of our system is the high degree of closure of flows. Thus, the hardest part it is to balance all technological processes. To solve this problem we suggest using rather complicated and accurate modelling founded on non-stationary partial differential equations describing the laws of matter conservation as a basis of an automatic control system. In this article, we present a mathematical model to simulate and control processes of transport phenomena in the system microalgae-mineralization-human. The model includes flow equations, the equations of reaction-advection-diffusion on a stratified set (linked 1D and 2D domains).

Keywords: Biosphere compatibility of cities; closed ecological life support systems; mathematical modelling; photobioreactor; treatment plants

1. Introduction

One of the biggest problems of megalopolises is the human organic waste disposal. According to the UN, the world volume of wastewater is about 1500 km\textsuperscript{3} per year, 80\% of which either has insufficient treatment or is discharged without treatment at all. In addition to the environmental aspect, the problem has significant economic consequences, since the electricity expended for water treatment in developed countries is between 2 and 3\% of the whole electricity that is produced.

Organization of human-waste treatment is one of the most important parts of the urban development projects. The current trend is to create treatment systems which are centralized as much as possible. Ideally, there should be no individual or small systems. It is believed that in this case the ecological situation can be fully controlled. However, such organization of process has side effects. Firstly, it is the possibility of occurrence of enormous environmental damage in case an incident at the central treatment plant. Secondly, the need for construction and maintenance of sewer lines of large length; some probability of defects of sewerage integrity and untreated sewage leakage. Thirdly, treatment facilities occupy large areas deforming the natural landscapes. Additionally, as mentioned, current treatment technologies are highly energy-intensive. Let us give some of the figures, which are typical for the average European city with population of about 500,000 people. Within the actual wastewater

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treatment technology, facilities for such a city are located on the square about 7 - 8 hectares, consume about 25 MW of electricity, and require maintaining 1,700 km of sewerage network.

We offer a new and daring concept of local treatment facilities (small objects with the ability to scale the volume that can be located in the immediate of buildings, and directly inside the building), the principle of which is based on a closed cycle biological transformation of matter. The main requirement such facilities meet is compliance with the principle of the biosphere-compatibility. The term Biosphere Compatible Economic Activity or Biosphere-Compatibility means an activity that uses technology based on the principles and laws of functioning of biological systems, principles of self-ecological communities, and closed circuit units of matter and energy.

We make a start from the experience of creation of the closed-loop biological life support systems (LSSs) that initially was being developed for long-term space flights. No other field of science has influenced imagination and vision more than the research and outcomes related to space exploration, either it is in terms of science fact, or science fiction. Analysis of the opportunities and prospects of using various space technologies for designing the systems on the Earth that may be described as being sustainable has been given by (Bock & Linner, 2010). In the 60s-80s of the last century, by a series of successful experiments with such kind of systems, the possibility to build an artificial closed ecosystem fully utilizing human waste was proved (Sychev, Levinskikh, & Shepelev, 2003), (Sychev, 2000). We develop the idea to apply this approach to the wastewater treatment systems for terrestrial conurbations.

2. The Project Concept

Obviously, the implementation of the idea of closed wastewater utilization systems similar to biological life support systems for space stations is associated with difficulties in terrestrial conditions. Firstly, traditionally, biological life-support systems are considered extremely energy-intensive. Secondly, balancing of the processes for small-volume facilities under conditions of randomly fluctuating wastewater volumes is a rather difficult task.

In the study to overcome these difficulties, we use the following key innovative approaches

- Implementation of closing system through oxidation processes, for example, combustion of residual biomass or using microbial fuel cells, what is possible to perform on Earth to create energy-efficient system;
- Balance control substances is performed by an automated control system, at the basis of which lies a direct mathematical modeling of biochemical processes in wastewater treatment plants, using the latest advances in the theory of constrained optimization in the form of partial differential equations.

During the 60s - 80s, at Institute of Biomedical Problems and Institute of Biophysics of Russian Academy of Sciences, there were several experiments with ecological LSSs based on the following links of the ecological chain: microalgae - mineralization - human and microalgae - mineralization - higher plants - human (Sychev, Levinskikh, & Shepelev, 2003), (Sychev, 2000). As the project being implemented currently, should be mentioned the MELiSSA -project (Micro-ecological life-support system) of the European Space Agency. In this project, scientists plan to include one more heterotrophic link – animals, namely fishes (Nelson, Pechurkin, Allen, Somova, & Gitelson, 2010).

In all types of closed ecological LLSs, algae is a basic element (link) of LLS, which allows regenerating the air, and water completely and provides (Sychev, Levinskikh, & Shepelev, 2003), (Sychev, 2000):

- Partial nitrogen cycle by way of the full use of human urine nitrogen by unicellular algae;
- Cleaning the atmosphere of a hermetic volume from various water-soluble gaseous contaminants through their full absorption and utilization in a photo-reactor by algae and attendant microorganisms (photo-reactor is an universal resettable hydro-biological filter);
- Optimization of air ion and aerosol composition of the atmosphere;
- Stabilizing of content of water-insoluble impurities (methane, carbon monoxide, etc.) by way their adsorption on the algae cell and microorganism surface and the subsequent removal of the grown biomass from the system;
- Ousting microflora, including pathogenic to humans, from micro biocenosis by a competitive relationship.
For our task, the structure Microalgae-Human-Mineralization option has been chosen as central and controllable core of system. In this case, it is possible to consider our treatment plant as a set of artificial aquatic ecosystems. Components of aquatic ecosystems is not different functionally from the components of terrestrial ecosystems. However there are some features. Organisms in the water biochemically are closely related to their environment and depend on the concentration of soluble substances. Due to the water density, which is the considerably greater than of air, many aquatic organisms are free-floating. The water consists of spatial-distributed suspended substances, microalgae, and microbes. The water also creates the possibility of biochemical links between communities of hydrobionts through the allocation of many organisms oxygen, carbon dioxide and various metabolic products into the water. These substances, toxic, or, on the contrary, stimulating other organisms, form the network by which organisms are connected implicitly, without direct contact. This feature of aquatic ecosystems allows describing their processes quite efficiently via the laws of matter conservation in the form of Reaction-Advection-Diffusion equations. As a result, we have an effective tool to create a reliable system for monitoring and control.

The functional diagram of the model Microalgae-Human-Mineralization is shown in Figure 1. The diagram has a simplified form but reflects the main streams of matter.

Stability and reliability of the structure Microalgae-Human-Mineralization is implemented by controlled flow circulating between a bioreactor-mineralizer and photoreactor cultivating algae. This combination of systems implement the natural self-purification processes, but which are intensified many times over. The structure can be integrated into a residential building in the form of the following combinations of elements presented in Figure 2.

Organic substances are oxidized in the mineralizer 3, mainly due to oxygen, which is formed in the photobioreactor. Thus, the key unit, the point of control of purification process’ efficiency is the flow distribution system, which could regulate flow rate of oxygenated liquid returning into the system i.e. coming from the device 2, photoreactor cultivating algae, to the device 4, bioreactor-mineralizer.

There is also possibility of having two extra controls in a system. The first is blowing the additional airflow into the bioreactor-mineralizer in some dynamic regime. For example, it is necessary to supply oxygen to bacteria in the dark, when there is no enough oxygen produced by algae. The second is flue gases injecting into the photoreactor for supplementary bacteria nutrition. Injection of air and flue gases can be carried out through saturators, devices for dissolution. Separation of sludge biomass occurs in a desilter: one part goes back into the reactor, and the excess is removed from the system. Residual biomass can be used as raw materials for the production of electricity through some oxidation process to close system. There can be different approaches (methane-tanks, biodiesel, combustion, composting, microbial fuel cells). Everyone has its own pros and cons. The main thing that the oxidation generates energy and CO2, which are used into the system then again.
3. Simulation and Control of Microalgae-Mineralization-Human Structure

Dynamics of elementary volume biochemical reactor can be written in vector form using the laws of conservation of mass and momentum as an artificial ecosystem (Astrakhantsev, Rukhovets, & Menshutkin, 2003), (Astrakhantsev & Rukhovets, 1994), (Bulgakow, Buzalo, Zhmenya, & Zaharchenko, 2012), (Buzalo, Zhmenya, & Ermachenko, 2014)

\[
\frac{\partial C}{\partial t} + U \cdot \text{grad}(C) - \text{div}(D \text{ grad}(C)) - W = 0 ;
\]

\[W = N \mu ;\]

\[
\frac{\partial U}{\partial t} + U \cdot \text{div}(U) - \text{div}(\nu \text{ grad}(U)) + \frac{1}{\rho} \text{grad}(P) = G ;
\]

\[\text{div}(U) = 0 ;\]

where \( C = (C_1, C_2, \ldots, C_{m+n}) = (S_1, S_2, \ldots, S_n, X_1, X_2, \ldots, X_p) \) is the vector of concentrations of substances which includes elements describing concentrations of biogens \( S_i \) (BOD, COD, ammonia nitrogen, nitrates, phosphates, oxygen, carbon dioxide, cell quotas) and concentrations of different kinds of microorganisms \( X_j \) (different kinds of microalgae); \( N \) is the matrix of stoichiometric coefficients, \([m+n \times l]\), where \( l \) is the number of processes leading to changing of concentration; \( \mu \) is the vector of the rates of biochemical reactions; \( W \) is the reaction term; \( D \) is the vector of diffusion coefficients; \( U \) is the flow velocity vector; \( P \) is pressure; \( G \) is the vector field of mass forces; \( \nu \) is viscosity, \( \rho \) is density.

Determining the reaction function has always been a complicated and debatable issue. There are several classical approaches to the description of the reaction member. Namely: Monod functions (Monod, 1949), which are suitable for chemostat-like systems where growth rate of microorganisms is limited by the substrate concentration; more complex functions taking into account the concentration of intracellular resources (Droop, 1974), (Jorgensen, 1976). Nowadays, for dynamic modeling of biological treatment facilities (mineralisers), different software is used (GPS-X, WEST, STOAT, etc.), which usually is built on standard models with activated sludge ASM (Active Sludge Model) (Henze, Harremoes, la Cour Jansen, & Arvin, 1995), (Henze, Gujer, Mino, & van Loosdrecht, 2000). However, it should be noted, that at the moment, there are no universal approaches and the methods described above can apply only to a certain, narrow class of problems. We believe that it is appropriate to use a linear combination of radial basis functions for reaction term setting. According to the principle of "limiting factors" (Liebig, 1840), and the law of "cumulative effects of factors" (Mitscherlich, 1909), (Mitscherlich, 1925). For every living organism, there is an environmental niche outside of which its inhibition or death occurs. In other words, the biocenosis state is determined by the sum of radial basis functions of wellbeing in the phase space of environmental factors

\[
\mu_i = \sum_j \mu_{ij}^\text{max} \cdot X_j \cdot \prod_{S_k \in \Omega_{X_j}} \exp \left( - \frac{(S_k - S_{ij}^0)^2}{\sigma_{ij}^2} \right), i = 1, 2, \ldots, l ;
\]

where, within the \( i^{th} \) biochemical process, \( \mu_{ij}^\text{max} \) is the maximum growth rate of the microorganism \( X_j \); \( \Omega_{X_j} \) is the set of substances affecting change in the concentration of microorganism \( X_j \); \( S_{ij}^0 \) is the optimum value of the concentration \( S_k \) which provides the maximum growth rate of the microorganism \( X_j \); \( \sigma_{ij} \) is the radius of tolerance of \( X_j \) to \( S_k \).

The form (5) has a fairly clear physical meaning and also it allows embedding a subprocess of data assimilation in the algorithm solving the problem (1 - 4) for correction of \( W \) by the use of radial basis neural networks. What is possible to do on a base of an additional software as well as with using a specialized hardware-chip. It is also important to note that the proposed mathematical model takes into account the inhibition of enzymatic processes, and therefore, is applicable for describing the trigger process (Ermachenko, 2012).
In the final version of the model, the flow equations in some elements of system can be reduced to Saint-Venant equations. The mathematical model is formulated as a boundary value problem defined on a stratified set containing 1D and 2D. The problem is supplemented by the initial, boundary and gluing conditions that we do not give because of their bulkiness. Since a technological solution for architecture of elements 2 and 4 depicted in Figure 2 has a variety of options the computational domain may take a different structure. One of the variants is shown in Figure 3. The Figure 4 shows some examples of calculations of velocity and substance concentrations in the reactor-mineralizer.

![Figure 3. Computational domain](image)

![Figure 4. Calculations of velocity and substance concentrations in the reactor-mineralizer (circulating oxidation ditch)](image)

The objective function for the control problem is minimizing of energy costs for air additional injected into the system through the saturator, and operation of the pumps and mixing devices. All of these control parameters are ‘source terms’, right-hand sides of either equations or boundary or gluing conditions. Denote their power as vector $f$. The time axis is divided into sections and controls are assumed as piecewise-constant. Then the optimization problem for each interval will have the following form

$$
\min_{f \in F_{ad}} J = \langle P, f \rangle;
$$

where $P$ is vector which components are corresponding of elements of vector $f$ and equal to one in sources areas, and zero – otherwise; $\langle \cdot, \cdot \rangle$ is the scalar product of a selected function space; $F_{ad}$ is the set of admissible controls. The constraints are, firstly, equations (1 - 4) with initial, boundary and gluing conditions, secondly, is constraints on water quality in significant areas (at the liquid's exit point from each of the devices). Thus state variable constraints are
\[ \{p_{k_i}, C_i\} \leq \Theta_{k_i}, \quad i = 1, 2, ..., m+n; \]  

where \( p_{k_i} \) are functions equal to one in \( k_i \)th significant area, and zero – otherwise; \( \Theta_{k_i} \) are given values.

To solve control problem (1 - 7) we use method of adjoint equations with linearization of the initial problem. As a result, our problem is reduced to a series of linear programming problems, which allows us to significantly reduce the problem and perform almost real time control despite the complexity of underlying model. We have described some features of this method of adjoint equations [7, 8].

6. Conclusion

The paper develops the idea of closed biological wastewater utilization systems. This work represents, to the authors’ best knowledge, the first application of theory of optimal control of matter and momentum conservations laws formulated as partial differential equations to problem of regulation of work of closed-loop treatment facilities. Further, a controlled closed-loop purification systems can be integrated in a building through the self-sustaining, interchangeable and standardized platforms, so-called mainboard-inspired one, situated beneath a modular home and controlling all water installments and energy components needed for a household, that being developed by (Bock & Linner, 2010). In conclusion, as the future direction of development of these complex of technologies, we would like to note such methods providing a mathematical model by data as the use of microfluidic sensors for chemical analysis and genome sequencing technologies that are actively developed in recent times.

References


