Abstract

The article addresses the problem of specifying the construction works duration time. Apart from the common deterministic method of schedule preparation, there are other, not so well-known methods. The type of the scheduling method is, in turn, related to the method of specifying the duration of construction works. The article briefly characterizes scheduling methods and the related duration times of construction works, such as: deterministic, probabilistic, fuzzy or combined (fuzzy-probabilistic). Simple examples illustrate the manner and form of activity duration values in different scheduling methods.

Keywords: schedule; scheduling methods; works duration time; probabilistic modelling; fuzzy logic

1. Introduction

Plans of the completion of investment projects are included in the construction organization plan, usually in the form of schedules which graphically depict the organization and technology of the implementation of an investment project in the time function. When preparing a schedule, one needs to consider every stage of its creation and make all the necessary decisions in order to gain an optimal effect as close to reality as possible. The issue of estimating the tasks completion timings belongs to these elements which may cause the most uncertainty at the construction completion planning stage. For this reason this issue was assumed as the critical element in the process of creating a schedule of work progress.

The article addresses the problem of specifying the construction works duration time. This may be achieved by the following methods: deterministic, probabilistic, fuzzy or combined (fuzzy-probabilistic). Examples of the use of the selected methods are presented, as well as certain restrictions and use conditions are discussed at the end of this paper.

Determining the time of construction works completion is closely related to the method of scheduling the project implementation adopted here. For this reason, the analysis described in the article, which determines the time of works completion, is supported by an analysis of scheduling methods.

2. An overview of deterministic scheduling methods

A study by [6] presents the following classification of issues related to the deterministic scheduling of construction works.
• Scheduling the project without taking into account resource constraints and without variants of ways to perform the activity (UPS – Unconstrained Project Scheduling).
• Scheduling the project including resource constraints but without variants of ways to perform the activity (RCPS – Resource Constrained Project Scheduling).
• Scheduling the project including resource constraints and variants of ways to perform the activity (MMRCPS - Multi-Modal Resource Constrained Project Scheduling)

Deterministic methods of scheduling include the following [6]:

• Scheduling by means of a distribution and limitation method;
• Scheduling with the use of priority heuristics;
• Scheduling with the use of metaheuristic algorithms, such as search with prohibited movements, simulated annealing or genetic algorithms.

In deterministic methods, the time of construction works implementation is typically calculated on the basis of the known amount of works (expressed by a work unit, for instance, m, m², kg, etc.) and the predicted work productivity (expressed as a work unit per time unit, for example, in meter per hour) [4, 10].

According to [2, 10], the duration of an activity may also be specified on the basis of historical data concerning specific activities and their average duration. Since it is not very probable that the scope of works is identical in various projects, additional performance indicators are used. Duration then can be specified in the following way:

\[ D = \frac{A}{P \times N} \] (1)

where: D – duration of an activity; A – the amount of a particular construction work measured in a relevant work unit; P – the average performance of an average brigade for a specific construction work (measured in a work unit per time unit); N – number of brigades assigned to complete a construction work.

Another possibility is the use of performance standards, which is the inverse of the standard time P:

\[ D = \frac{S \times A}{N} \] (2)

where: S – time needed to finish a work unit by an average brigade (measured in a time unit per work unit).

[8] presents an analogous way of calculating the duration of a task:

\[ t_i = \max_{l \in S_i} \frac{p_{i,l}}{a_{i,l}} \] (3)

where: p_{i,l} – effort involved in the i-th task in relation to the l-th resource; a_{i,l} – number of active resources of l-th type assigned to complete the i-th task; S_i – a set of active resources for the i-th task.

Deterministic scheduling methods are the most common in construction practice. Calculations are based on deterministic values and it makes them understandable and easy to analyze for all contractors involved in the same project. All one needs are sources of data or basics to determine them.

3. Scheduling under uncertainty

The specificity of construction project completion is the reason for many elements necessary for planning to be random. Nowadays, one can observe two directions of developing scheduling methods for construction works under uncertain conditions of implementation.
• One of them involves methods in which the uncertainty about the conditions of works execution is modelled by means of a probability measure. A schedule is prepared assuming an estimation of the completion time of particular works equal to the expected values or to the median of their probability distributions.
• Another one concerns methods in which modelling of imprecisely specified relationships between the frequency and intensity of the interference and its consequences is based on the theory of fuzzy sets.

These methods are more time consuming and difficult to use in practice. A contractor need to have more experience and possibility to make observations and collect proper data. Calculations are more mathematically advanced and require more detailed knowledge about method applying rules. One the other hand, using probabilistic and fuzzy methods allows the uncertainty consideration.

3.1. Description of probabilistic scheduling methods

The first project scheduling method which involved a probabilistic modelling of random interference was the PERT method. PERT is commonly known as a method of a construction project analysis, in which activity duration is specified by means of probability beta distribution. Beta distribution is applied since it allows to specify the minimum and maximum time of the possible duration of an activity [4, 5]. Assuming that duration is a random variable \( t \), \( a \) indicates the (optimistic) minimum time and \( b \) is (pessimistic) maximum time, the density function for \( t \in <a;b> \) can be expressed by the following formula [5]:

\[
f(t) = \frac{1}{\beta(p,q)(b-a)^{p+q}}(t-a)^{p-1}(b-t)^{q-1}
\]  

(4)

where:  
\[
\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1} \, dx
\]

As a consequence, the distribution curve depends on the \( p \) and \( q \) parameters. A series of transformations are described in detail in [1]. The crucial point for this way of determining time is the fact that estimation by means of beta distribution requires three pieces of information: about the most optimistic (minimum), the most probable and pessimistic (maximum) time. The average (expected) duration \( t_e \) is calculated according to the formula below:

\[
t_e = \frac{a + 4m + b}{6}
\]  

(5)

where:  
\( a \) – optimistic time; \( m \) – the most probable time; \( b \) – pessimistic time.

To take account of the uncertainty in the estimation of works duration one may also apply other probability distributions. Duration can be considered a random variable with normal or triangular distribution. Normal distribution is convenient to use as it relies on two parameters (therefore it is frequently applied as an approximation of beta distribution). On the other hand, the greatest advantage of the triangular distribution is that it requires three unambiguously specified deterministic values: the minimum, the most probable and the maximum value of the random variable [7]. The density function for the triangular distribution for \( t \in <a;b> \) is presented below.

\[
f(t) = \begin{cases} 
\frac{2(t-a)}{(b-a)(m-a)}, & t \in <a;m) \\
\frac{2}{(b-a)}, & t = m \\
\frac{2(b-t)}{(b-a)(b-m)}, & t \in (m;b> 
\end{cases}
\]

(6)
where: \(a, m\) and \(b\) are defined exactly like in equation (5).

*PERT* became the basis of a method of schedule stabilizing which uses the criterion of network program reliability. Then, to model random interferences, simulation techniques were introduced. To address the unfavourable accumulation of floating times which appear as a result of the application of simulation techniques, a critical chain method was designed. The flaws of this method, in turn, became an incentive for the development of the predictive-reactive scheduling method involving the criterion of minimizing the cost of schedule instability. Table 1 presents probabilistic methods and the values of activity duration that they include.

Table 1. Probabilistic methods

<table>
<thead>
<tr>
<th>PROBABILISTIC SCHEDULING METHODS</th>
<th>GROUP I</th>
<th>GROUP II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method using the criterion of network program reliability</td>
<td>The critical chain method</td>
<td>Simulation method</td>
</tr>
<tr>
<td><strong>ACTIVITY DURATION VALUES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected values</td>
<td>Median</td>
<td>Expected values</td>
</tr>
<tr>
<td>Probability distribution</td>
<td>Standard</td>
<td>Individually determined for particular activities</td>
</tr>
</tbody>
</table>

Prepared on the basis of [6]

The characteristic feature of the first group of methods includes the fact that they ensure stability of the planned time of project completion. On the other hand, the second group is characterised by the possibility to ensure the stability of the planned beginning times of the project activities. Except for the simulation method, the methods described here consist of a two-stage preparation of a stable project schedule, yet each method involves a different way of ensuring such stability. This issue was described in detail in [6].

3.2. Description of fuzzy scheduling methods

The application of the fuzzy sets theory, as well as the relations between the probability distribution and the distribution of opportunities, allows to designate fuzzy numbers which model works completion times for the analysis of networks and fuzzy construction schedule. The existing methods of fuzzy scheduling belong to two groups.

One group includes the methods which are equivalent to methods scheduling projects of determined conditions of implementation. When there exists unlimited access to renewable resources, the specification of the earliest and the latest activity completion times depends on the relations known in the classic method of the project’s critical path analysis and the principles of fuzzy arithmetic. Then access to renewable resources is limited, ordering of activities involved in the project is based on fuzzy equivalents of optimization methods.

The other group comprises methods encompassing the concept of \(\alpha\)-intersections of the fuzzy number and the principles of interval numbers arithmetic. A fuzzy schedule is obtained as a result of assembling intervals computed for a finite number of \(\alpha\)-intersections of fuzzy numbers, which model the durations of individual activities in the network model of a construction project.

Fuzzy modelling of construction completion times includes the following:

- fuzzy modelling of works completion times based on expert opinion and
- modelling works completion times based on fuzzy reasoning.

A study by [12] presents a fuzzy equivalent of a division and constraint method using DSF strategies to solve the task of minimizing the project completion time span, taking into account the limited availability of renewable resources. Research into priority heuristics includes such studies as [3, 9, 13] which present schemes of parallel generation of a fuzzy schedule allowing for limited access to resources. Another study by [1] describes a method equivalent to *PERT*, which models the imprecisely specified times of activity completion with the use of triangular fuzzy numbers. On the other hand, [11] attempts to evaluate a fuzzy schedule by applying the measure of capability and obviousness to compare fuzzy numbers which model the planned and the accepted time of project completion.
3.3. Fuzzy-probabilistic methods of construction scheduling

The fuzzy-probabilistic methods by J. Kulejewski presented in Table 2 were described in detail in [6]. These methods include the diverse levels of knowledge that contractors possess concerning the accessibility of resources, the acceptable project completion time and the consequences of the anticipated interferences into construction works.

<table>
<thead>
<tr>
<th>FUZZY-PROBABILISTIC METHODS OF SCHEDULING</th>
<th>Method 1</th>
<th>Method 3</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unlimited access to resources</td>
<td>Access to resources is limited but the limitations are imprecisely specified</td>
<td></td>
<td></td>
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<tr>
<td>Time necessary to complete one project is limited due to the need to release the resources owned to complete another project, but the beginning time of this project cannot be precisely specified.</td>
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<td></td>
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</tr>
<tr>
<td>Interferences in construction works are anticipated but the inevitable consequences of the interferences cannot be precisely specified.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interferences in construction works are anticipated, but their consequences can be avoided due to careful prevention and control.</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>VALUES OF ACTIVITY COMPLETION TIMES</th>
<th>Fuzzy value</th>
<th>Deterministic value</th>
<th>Fuzzy value</th>
</tr>
</thead>
</table>

Based on [6].

The basis of these solutions is mapping the construction by a one-point relations network with one initial activity and one final activity. Between these activities there exists an end – beginning relation. When it is possible to predict the consequences of interferences, activity completion times are known and expressed by real numbers. Otherwise, estimations of activity completion times are modelled by fuzzy numbers and include reserves which allow for a fuzzy evaluation of the influence of interferences on the particular activity. The existing planning limitations are also modelled by fuzzy numbers.

Justification for referring to these methods as fuzzy-probabilistic, and not simply fuzzy, is that all three methods apply a probabilistic evaluation for the analysis of the level of compliance with the fuzzy time limit spent on construction implementation (previously, two probabilistic measures were used, that is the degree of obviousness and degree of capability). Moreover, the level of compliance with the fuzzy limitation to resources accessibility undergoes a probabilistic evaluation too (previously, only the deterministic aspect of these constraints was considered).

4. Examples of designating activity duration in schedules

Example 1. The deterministic method

Let us assume that for a given construction work (work A) the data are as follows:
- Time standard $S = 2,01$ man-hour/m³
- Performance standard for a 6-person brigade – $P = 2,985$ m³/man-hour
- Number of works – $A = 90$ m³
- Number of workers – $N = 6$

According to formula 1, duration can be computed as follows:
$D = 90 \text{ m}^3/2,985 \text{ m}^3/\text{m-h} \cdot 1 = 30,15 \text{ h}/8 = 4 \text{ days}$

In accordance with formula 2, duration equals:
$D = 2,01 \text{ m-h/m}^3 \cdot 90 \text{ m}^3/6 = 30,15 \text{ h}/8 = 4 \text{ days}$
Example 2. The Probabilistic method

To specify the expected time of work A, according to formula (5), one needs to calculate three duration values: the pessimistic, most probable and optimistic. Assuming that the most probable duration is the time computed in example 1:

- optimistic time $a = 3$ days;
- the most probable time $m = 4$ days;
- pessimistic time $b = 7$ days;

The average (expected) duration $t_e$ is calculated below:

$$t_e = \frac{3 + 4 \cdot 4 + 7}{6} = 4.33$$

The waiting time for work A assumed in the schedule will be 5 days.

Example 3. The fuzzy value

The fuzzy value of duration time can be computed similarly to formulas (1) and (2). Here all the data must be fuzzy numbers.

Let us assume that the standard and number of workers is a fuzzy number with a trapezoid membership function, and the amount of works is a fuzzy number with a triangular membership function.

Then the data for the designation of work A time are as follows:

- Standard $S_i$ - (1.98; 2.01; 2.33; 2.5) man-hour/m³
- Number of works $A_i$ - (80; 90; 140) m³
- Number of workers $N_i$ - (5; 6; 7; 8)

Time computed on the basis of the following formula:

$$T_i = S_i \cdot \frac{A_i}{N_i} = (s_{a1}/ n_4; s_{a2}/ n_3; s_{a3}/ n_2; s_{a4}/ n_1) = (t_1; t_2; t_3; t_4)$$

$$T_i = (1.98 \cdot 80/ 5/ 8; 2.01 \cdot 90/ 6/ 8; 2.33 \cdot 90/ 7/ 8; 2.5 \cdot 140/ 8/ 8) = (4; 4; 4; 5)$$

The duration value obtained is a fuzzy number with a trapezoid membership function.

5. Conclusions

One of the elements indispensable for the preparation of a construction works schedule are the durations of particular works. Their values directly influence the correct specification of the investment completion time. Apart from the common deterministic method of schedule preparation, there are other, not so well-known methods. The type of the scheduling method is, in turn, related to the method of specifying the duration of construction works.

The article briefly characterizes scheduling methods and the related duration times of construction works. Simple examples illustrate the manner and form of activity duration values in different scheduling methods. The main contribution of the paper is different methods advantages and disadvantages specification. Contractors knowing restrictions of each method would be able to choose this one which is the most appropriate for their problem.

References

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