Abstract

In this paper, we introduce a model and define an algorithm for the Facilities Location Problem with focus on optimizing the arrangement of final material depots at a construction site that is represented by a concave area. The available area of the construction site for placing the depots corresponds to the area of the built-in material location that is served from the depots. The model solves the problem by using the known Voronoi diagram. The goal is to minimise the sum of the delivery distances from the final material depots to each point of the structure (continuous problem) instead of the most models that minimise the sum of the delivery distances from the final material depots to the built in location of each element of the structure (discrete problem). This way the number and the locations of the built in material elements that define the structure are left out of consideration. The algorithm gives near-optimal solutions. Advantages of the model and the algorithm are presented based on an example. The applicability of the model are summarised. This model can be used as a tool for construction site layout planning if the number of units that define the structure is large or unknown.

Keywords: continuous demand; construction site elements allocation; construction site layout planning; facility location allocation problem; Voronoi diagram

1. Introduction

One of the preliminary processes in the construction phase of building development is planning the construction process. Part of the construction planning process is construction site layout planning (CSLP), in which space, time, material, labour, money and equipment are recognised as resources [1, 2]. The target of CSLP is to minimise construction time, cost or required resources. A part of CSLP is the construction site elements allocation. The site elements are categorised into three object groups: site, construction and constraint [3]. According to Moore [4], there are two basic methods that can be utilised to manage the CSLP problem. The first method is to place the collection of objects in all possible configurations (or a reduced subset of these combinations) and to choose the best of these solutions. The second method, which will be used in this paper, is to sequentially place objects based on a pre-defined order and to calculate the optimal arrangement after each step. One type of construction object is the final material depot where from the material is handled to each point of the structure by one piece after the other. The optimal arrangement of the final material depots (AFMD) of a type of material is where the sum of the handling distances is minimised if the handling technology is constant.
In operation research literature, the root of the AFMD problem is known as the \textit{k-median problem} and it is a part of the location allocation problem (LAP). In the operation research literature there are studies for the discrete type optimal location problems and for the continuous type optimal location problems as well. Most studies for the CSLP use the discrete version. The discrete version starts with a two-dimensional figure on the plane that consists \textit{m} predefined points inside the figure as possible hosts and selects \textit{k} of them as depots to minimise the total handling distance. A discrete version of the problem as an approximation to the continuous problem is demonstrated [5].

The Voronoi diagram can be used to minimise the average handling distance. In mathematics, a \textit{Voronoi diagram} (also called a \textit{Dirichlet tessellation}) is a method of dividing an area into a number of regions. An initial set of points (called seeds, \( S \) ) is specified, and for each seed there is a corresponding region consisting of all of the points that are closer to that seed than to any other seed (also known as host). These regions are called Voronoi cells [6]. Voronoi regions can be produced by connecting the centres of the circumcircles of the Delanuay triangulation [7].

In this study, we present an algorithm based on Voronoi diagram and used the continuous model of LAP that results optimal arrangement of the final material depots for one type of material. In cases in which the capacities of the depots have to be equal and the handling paths must be inside the two-dimensional figure and the final material depots can be positioned inside the served working area (inside the Voronoi cells) and the final locations of the built-in material pieces are unknown in advance.

\section*{2. Assumptions and objective functions}

Architects and engineers define most structures with 3D CAD elements. The structures are represented by 2D marking with the Z directional information included on the drawings. Some structures are marked by symbols (e.g. pillars or windows), some are marked by line segments (e.g. wall tiling) and some are marked by areas (e.g. floor tiling, concrete slabs or the boarding of the slab formwork). The material, dimensions, volume and exact location of the structure are given in the architectural documentation. The area of the structure is a two-dimensional figure denoted by the end points (\( A_i \)) of the line-segments that are bounding the area.

\subsection*{2.1. Assumption 1.}

The final material depots are represented by the projection of their centre of gravity \( S(x, y) \) to the \( XY \) plane. One type of material depot usually consists of a certain number of material elements resulting in equal material depot volumes. The number of the required depots (\( k, k \in \mathbb{N} \)) can be easily calculated by dividing the volume of the structure by the volume of the final material depots. We assume that each final material depot should serve equal sized subarea of the two-dimensional figure or at least the maximal deflection of the sizes among the sub-areas (\( D, D \in \mathbb{Q} \)) should be given in advance.

\subsection*{2.2. Assumption 2.}

There are technologies where the final depot can be placed into the area that is served from the certain depot (inside the subarea of the two-dimensional figure) and there are technologies when the final depot must be placed out of the served area (outside of the subarea of the two-dimensional figure) [8]. In this study we assume that the depots must be inside the served area and the available area is the two-dimensional figure.

\subsection*{2.3. Assumption 3.}

The handling paths from a depot to each point of the served subarea can be calculated by two ways: using Euclidean distance or the shortest path inside the two-dimensional figure. In this study we assume that the handling paths must be inside the two-dimensional figure.

\subsection*{2.4. Objective function 1.}

The objective is to find the allocation of the final material depots, where from the material can be handled by pieces to their built-in locations along the minimal length of paths.
2.4.1. In the case of \( k=1 \) (discrete model)

The problem is known as the classical Fermat-Weber problem \(1929\) [9], and the solution can be found by any two-parameter minimization. If the built-in locations of each pieces of the material are given in advance and the number of the pieces is not extreme large, than the discrete model can be used:

\[
\min_S \sum_{i=1}^{n} d_{IS} \tag{1}
\]

where \( n \) is the total number of the elements and \( d_{IS} \) is the handling distance from the location of the material depot \( S \) to its built-in location \( i \).

2.4.2. In the case of \( k=1 \) (continuous model)

If the dimensions of the elements contained in a depot are extreme small, (or the built-in locations of the pieces of the material are unknown) then the equation turns into a double integral (Eq.2.) that is also a volume integral. This is the continuous model that is used in this study:

\[
\min_S \int_{R} \int_{S} (d_{IS}) dxdy \tag{2}
\]

In Eq. 2., where \( (R) \) is a bounded and closed region of the \( XY \) plane; \( S \) is the location of the material depot; \( d_{IS} \) is the handling distance from the material depot location to each point of the figure \( (i) \).

2.4.3. In the case of \( k>1 \) (discrete model)

The goal of the discrete model is:

\[
\min_S \sum_{j=1}^{k} \sum_{i=1}^{n} d_{IS_j} \tag{3}
\]

where \( k \) \((j=1\ldots k, k \in \mathbb{N})\) is the number of final material depots.

2.4.4. In the case of \( k>1 \) (continuous model)

For the continuous model the number of the variables in the equation increases:

\[
\min_S \sum_{j=1}^{k} \int_{R} \int_{S_j} (d_{IS}) dxdy \tag{4}
\]

The locations of the final material depots \( (S) \) are variables and the boundaries of the subareas are also variables that are served by the depots. In this equation the number of variables is too many. In case of the two-dimensional figure would be partitioned into \( k \) pieces of equal size subarea than the final material depots could be searched for each subareas by using Eq.2. for each. We use a modified version of the well-known method named Voronoi regions for the partitioning.

2.5. Assumption 4. Hypothesis

If \( k>1 \) then there are infinite solutions to partition the area of the structure into \( k \) equal size subareas (called cells). Our experimental solution is that the partitioning will result the minimal sum of the delivery distances where the cells have the minimal length of the perimeter sum (Eq.5.). This so far has not improved yet the demonstration of it will be published.
$$\min_{S} \sum_{j=1}^{k} p_j$$  (5)

2.6. Assumption 5.

If two points $S_1, S_2$ are set as seeds in a concave two-dimensional figure, then the calculated Voronoi cells often result in disconnected cells. We predefined that we use the shortest path inside the polygon, not the straight-line motion for the handling of the material (assumption 3.). In these cases, the polygon of the two-dimensional figure should be used as an obstacle. In case the common boundary of the Voronoi cells crosses the boundary of the polygon represents the available area more than two times the bounding of the voronoi cells must be modified. The inside area of the two-dimensional figure should be divided into areas that are “visible from” and “not visible from” both seeds and “invisible from one seed” based on Sadeghpour et al. [3, 10].

![Figure 1.a. Voronoi cells](image1a) ![Figure 1.b. modified Voronoi cells](image1b)

The common boundary of the modified Voronoi cells is a hyperbola or partly a hyperbola and partly a line-segment shown on Figure 1.b. The difference between the Voronoi cells in case of the handling paths can be calculated by Euclidean distance (Figure 1.a.) and in case of the handling paths can be calculated by the shortest path inside the two-dimension figure (Figure 1.b.) is shown. The green line is a part of the original bounding of the Voronoi cells and the blue line is the modified part of the bounding of the cells (hyperbola).

3. The model

3.1. Given in advance

The architects and engineers defined the area of the structure that should be built as a two-dimensional figure. Let it be a polygon shown on Figure 2. The order of the needed objects (type of the materials) should be predefined. The model deals with one type of an object at a time. The volume of the two-dimensional figure and the volume of a depot are also given in advance. The exact built-in locations and the number of the needed pieces of a material are unknown.

3.2. Searched

The locations of the final material depots are searched, where from the sum of the handling distance is minimal.

3.3. Algorithm

The needed number of the depots ($k>1$, $k \in \mathbb{N}$) can be calculated by dividing the volume of the two-dimensional figure with the volume of a depot. A set of points ($q$) inside the polygon are set ($q \in \mathbb{N}$ and $q>k$ ) where from $k$ points are selected as seeds (Figure 2.a.). The set of $q$ points is practical to be on a grid. For all permutation of $q$ points that contain exactly $k$ elements, the Delaunay triangulation should be performed. In all cases where one side of the Delaunay triangle crosses the concave polygon structure, the area of the structure should be divided into visible from and invisible from areas for those two seeds, and the bounding of the cells should be modified as described in Assumption 5. (Figure 2.b.). For those points where the Delaunay triangle does not cross the polygon, the areas
should be treated as Voronoi regions. From all of the permutations of \( q \) points, as described previously, where exactly \( k \) seeds are selected the perimeter of the cells and the area of the cells should be calculated. In assumption 1. it was set that all depots have equal capacities so each cells should have equal area size, but the Voronoi regions (cells) rarely have equal areas because the result is based on the allocation of the pre-set \( q \) points. Based on Assumption 1. and Assumption 4. a Precision value (Eq.6.) compares each result of the permutation with each other:

\[
\text{Precision} = \frac{\text{Min}[\text{areas}]}{\text{Max}[\text{areas}]} \sum_{j=1}^{k} p_j
\]

where \( \text{Max}[\text{areas}] \) is the maximum cell area, \( \text{Min}[\text{areas}] \) is the minimum cell area and \( p_j \) is the perimeter of the cells.

From all of the permutations of \( q \) points, where exactly \( k \) seeds are selected only that result should be recorded where the Precision has the maximum value. If the cell areas are not equal in size or we want to increase the preciseness of the result, then the results should be recalculated by setting up a new set of \( q \) points around the recorded seeds. It can be repeated until the difference between the minimum and the maximum cell sizes is negligible (Figure 2.c.).

![Figure 2.a. Points are selected as seeds (k=4)](image1)

![Figure 2.b. Modified cells](image2)

![Figure 2.c. New set of q points](image3)

Once the partitioning of the area is done the locations of the final material depots can be searched for each cell by using Eq.2.

### 4. Analysing the methods by an application

In this example, we use Wolfram Mathematica 7. The structure is a concave two dimension figure defined by the following polygon break points: \( A_1=(0,0), A_2=(5.641,3.257), A_3=(-0.505,6.535), A_4=(2.018,4.726), A_5=(-0.566,2.301), A_6=(2.609,3.274) \). Let \( k=4 \) seeds. At first, we calculated the solutions where all the handling paths are straight-line movements (the carrying paths may use areas out of the perimeter) and the cell areas do not need to be equally size areas. The results are shown in the second column of Table 1. as original model and the Voronoi regions are presented in Figure 3.a.

Next, we calculated the solution when all the handling paths must be inside the polygon. The feasible locations of the seeds were the same as the original models set. The results of the first, second and fifth iterations are marked in the third, fourth and fifth column of Table 1. as our model (Figure 3.b.). In this calculation, we limited the number of algorithm iterations to five so the best solution is shown on the fifth column of Table 1. and on Figure 3.b.3.
Table 1. Results of the model

<table>
<thead>
<tr>
<th>Locations of the seeds (S)</th>
<th>Original model</th>
<th>Our model (1st iteration)</th>
<th>Our model (2nd iteration)</th>
<th>Our model (5th iteration)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>{1.242,1.240}</td>
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<td>{2.048,1.641}</td>
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<tr>
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<td>{4.202,3.488}</td>
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<td>{4.229,3.076}</td>
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<td>{2.492,4.630}</td>
<td>{2.715,4.200}</td>
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<tr>
<td></td>
<td>{1.741,4.314}</td>
<td>{1.217,3.281}</td>
<td>{1.253,3.418}</td>
<td>{1.901,3.328}</td>
</tr>
<tr>
<td>Areas of the voronoi cells</td>
<td>1.953</td>
<td>2.514</td>
<td>3.138</td>
<td>2.773</td>
</tr>
<tr>
<td></td>
<td>2.621</td>
<td>3.134</td>
<td>2.628</td>
<td>2.772</td>
</tr>
<tr>
<td></td>
<td>3.510</td>
<td>2.095</td>
<td>2.186</td>
<td>2.651</td>
</tr>
<tr>
<td>Sum of the perimeters of the cells</td>
<td>50.212</td>
<td>31.49</td>
<td>28.82</td>
<td>38.29</td>
</tr>
<tr>
<td>Precision of the solution</td>
<td>0.011081220</td>
<td>0.01927636</td>
<td>0.02337283</td>
<td>0.02338223</td>
</tr>
</tbody>
</table>

5. Results and Discussion

In this paper, a model and an algorithm were presented to address a Facilities Location Problem in which construction sites have a concave shape and all of the handling paths must be inside the polygon of the structure. None of these two models gives the global optimal solution for the original objective but the results can be as precise as the certain technology needs. In case of the pre-set grid as feasible location of the seeds is too large the calculation may stick into a local optimal solution. In this model, the delivery cost was not considered because the primary focus was the arrangement of the final depots for one type of a material; however, this model can also be integrated with any CSLP model to minimise the total delivery cost.

References